

HUMAN POPULATION AND CARBON DIOXIDE

W. M. Schaffer

Department of Ecology and Evolutionary Biology

The University of Arizona

Tucson, AZ 75721

wms@u.arizona.edu

Abstract: A recently proposed model of human population and carbon utilization is reviewed. Depending on parameter values, one of three possible long-term outcomes obtains. 1. Atmospheric carbon, $[\text{CO}_2]_{\text{atm}}$, and human populations equilibrate at positive values. 2. The human population stabilizes, while $[\text{CO}_2]_{\text{atm}}$ increases without bound. 3. The human population goes extinct and atmospheric carbon declines to 0. The final possibility is qualitatively compatible with both “consensus” views of climate change and the opinions of those who argue against it.

In a recent communication to this journal, Lonngren and Bai (2008) proposed what a student of biochemical kinetics would refer to as an “abstract” model (LB) of human population growth and anthropogenic climate change. Phenomenological models, because they omit complicating factors, are incapable of quantitative prediction. At the same time, their cartoon-like simplicity focuses attention on the important task of identifying mechanistic essentials. They are also amenable to mathematical analysis, with the consequence that they can often be understood in detail. By way of contrast, elaborate simulations run the risk of replacing poorly understood systems with models that are likewise opaque (Borges, 1998). This having been said, both the analysis of LB and its derivation warrant comment. The purpose of this note is three-fold: first, to clarify the model’s dynamics; second, to reconsider its assumptions and finally, to suggest a somewhat more general scheme for contemplating the coevolution of carbon and climate. By way of conclusion, we note that LB’s most noteworthy prediction – human extinction and no atmospheric carbon (more accurately, *anthropogenic* carbon) – is qualitatively consistent with both the consensus view of global warming and the opinions of those who argue against it.

The LB Model. The equations proposed by Lonngren and Bai are as follows:

$$\begin{aligned}\dot{P} &= \alpha P - \beta PC + \gamma C \\ \dot{C} &= -\delta C + \varepsilon CP\end{aligned}\tag{1}$$

Here superscripted dots signify time derivatives. Additionally, P is the human population; C , the concentration, $[\text{CO}_2]_{\text{atm}}$, of atmospheric CO_2 and the quantities, α , β , δ , ε and γ , positive parameters. Note that Equations (1) replace the term, $-\gamma C$, in the original with $+\gamma C$. This reflects the fact that Lonngren and Bai set $\gamma = -.01$ in the course of implementing their equations numerically.

LB Dynamics. Equations (1) have two equilibria – a trivial equilibrium $(P, C) = (0, 0)$ and a nontrivial equilibrium, (P^*, C^*) at

$$P^* = \delta / \varepsilon; \quad C^* = \frac{\alpha(\delta / \varepsilon)}{\beta(\delta / \varepsilon) - \gamma}\tag{2}$$

Of these, the origin, $(0, 0)$ is a saddle, with eigenvalues, $\lambda_1 = -\delta$; $\lambda_2 = \alpha$.

Positivity of (P^*, C^*) requires

$$\xi = \beta(\delta / \varepsilon) - \gamma > 0,\tag{3}$$

which is also the condition for local stability. More precisely, the eigenvalues are roots of the characteristic equation

$$\Delta(\lambda) \equiv \lambda^2 + \frac{\alpha\gamma}{\xi}\lambda + \alpha\delta = 0. \quad (4a)$$

Satisfaction of condition (3), *i.e.*, $\xi > 0$, guarantees that the roots of Equation (4a) are negative, if real, or that they have negative real parts, if complex. Conversely, $\xi < 0$ implies that the roots of Equation (4) are real positive, or have positive real parts. In either case, the condition for complex roots, and hence for oscillations, is

$$\xi^2 > \frac{\alpha\gamma^2}{4\delta} \quad (4b)$$

It is useful to compute the zero-growth isoclines (null clines) of Equations (1). These obtain by setting the time derivatives equal to zero. Thus (Figure 1),

$$\dot{P} = 0 \Leftrightarrow C(P) = \frac{\alpha P}{\beta P - \gamma} \quad (5)$$

$$\dot{C} = 0 \Leftrightarrow P = \delta / \varepsilon$$

Note that in the limit of large P , $C(P) \rightarrow \alpha / \beta$. Also note that the P -isocline diverges at $P = \gamma / \beta$. As a result, there are two branches, only one of which is shown in the figure. The other branch is in the 4th quadrant and traces from the origin to $(\gamma / \beta, -\infty)$.

Zero-growth isoclines inform one's understanding of motion – more technically, the “flow,” (Guckenheimer and Holmes, 1983) – in the phase plane. In the present case, trajectories to the right of the C -isocline move up, while those to the left, move down. Correspondingly, trajectories above the positive branch, *i.e.*, for $P > \gamma / \beta$, of the P -isocline move left, and those below, to the right.¹ One can thus populate the P - C plane with vectors indicating the joint motion of the two variables. The reader can confirm that the sample trajectories in Figure 1 conform to the “direction field” so constructed.

As shown in Figure 1, the dynamics induced by Equations (1) depend critically on whether or not condition (3) is satisfied. When it is (Figure 1a), the system settles down to (P^*, C^*) . If condition (4b) is also satisfied, the approach to equilibrium is via damped oscillations, in which case the period, T , near equilibrium is given by

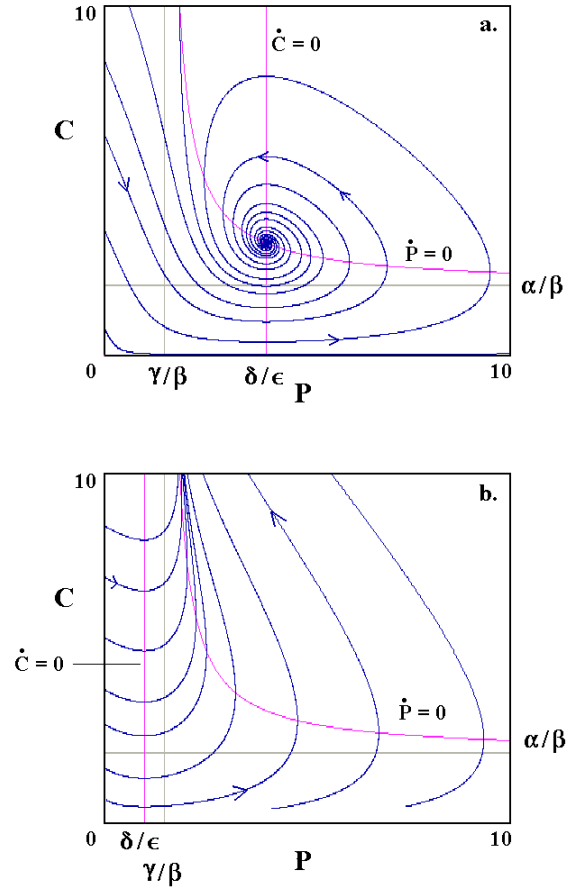


Figure 1. Zero growth isoclines and representative trajectories for Eqs. (1). For both calculations: $\alpha = 0.2$; $\beta = 0.1$; $\varepsilon = 0.1$. **a.** $\gamma = 0.15$; $\delta = 0.4$. **b.** $\gamma = 0.15$; $\delta = 0.1$. In **a.**, $\xi = \beta(\delta/\varepsilon) - \gamma > 0$. In **b.**, $\xi < 0$.

¹ For $P < \gamma / \beta$, the situation is reversed. Trajectories above the left branch of the P -isocline move right; those below, left.

$$T \approx \frac{2\pi}{4\alpha\delta - (\alpha\gamma/\xi)^2}. \quad (6)$$

This is the case reported by Lonngren and Bai. Conversely, when condition (3) is violated (Figure 1b), trajectories follow the ascending branch of the P -isocline. In the limit of large time, $(P, C) \rightarrow (\gamma/\beta, \infty)$ – *i.e.*, the human population tends to a fixed value, while atmospheric carbon grows without bound.

Negative γ . Lonngren and Bai observe that with $\gamma = 0$, Equations (1) are formally equivalent to the model of Volterra (1926), with atmospheric carbon playing the role of predators, and the human population, that of their victims. Then, the non-trivial equilibrium is a center, and the P - C plane, densely filled by an infinite number of neutrally stable periodic orbits.²

With the addition of the term, $+\gamma C$, the model becomes something else entirely. Now the “predators,” in addition to eating “prey” in proportion to βPC , augment the latter’s growth rate in a manner that is independent of their density. The number and variety of ecological interactions being effectively limitless, one can conjecture possible examples. For example, hooved mammals consume plants (negative effect) in rough proportion to the multiplied densities of consumer and consumed. At the same time, they fertilize the soil with their droppings (positive effect). If one imagines vast numbers of such creatures moving continually across large areas – think bison, wildebeest, *etc.* – this enrichment will be independent of the local density of forage and thus solely in proportion to the size of the herd.³ Such mixed interactions are not without interest. But they are a long way from the paradigmatic example of “rabbits and foxes.”

What about negative values of γ , the analog of bison trampling prairie grasses into the ground? The consequence of this change is shown in Figure 2. In this case, trajectories leave the vicinity of (P^*, C^*) , either directly or via oscillations of increasing amplitude – the case shown. Eventually, P goes negative. Thereafter, the saddle at the origin shoots trajectories off to $(P, C) = (-\infty, 0)$.

To summarize this and the preceding section, we have learned three things:

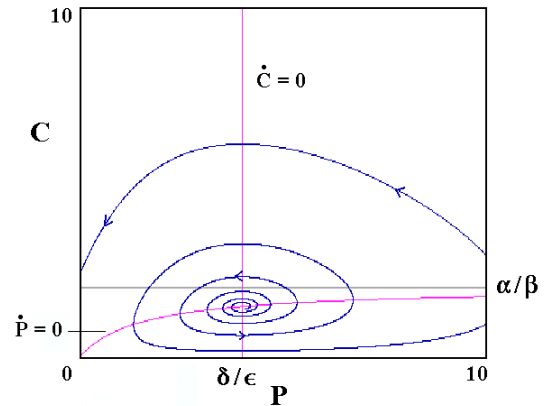


Figure 2. Zero growth isoclines and representative trajectories for Equations (1) with $\gamma < 0$. Here, $\alpha = 0.2$; $\beta = 0.1$; $\gamma = -0.15$; $\delta = 0.4$ $\epsilon = 0.1$.

² One way of establishing this is to observe (King and Schaffer, 1999) that, under a suitable transformation of coordinates, the equations are Hamiltonian.

³ The analogy’s legitimacy thus depends on the ratio of migratory to digestive time scales.

1. With $\gamma \neq 0$, LB dynamics are quite different from those observed in conventional predator-prey models.⁴
2. With γ positive, the human population stabilizes at a positive value, while $[\text{CO}_2]_{\text{atm}}$ either stabilizes or increases without bound.
3. With γ negative, the human population goes extinct ($P \rightarrow -\infty$), and $[\text{CO}_2]_{\text{atm}}$ drops to zero.

The final possibility reminds us that C is best thought of as anthropogenic, as opposed to total, CO_2 .

Derivation of the Model. Whether or not Equations (1) are usefully viewed as a modified predator-prey system is of interest primarily to those with affection for such models in their traditional context. To assess their relevance to human populations and climate, it is necessary to consider the reasoning and assumptions in back, so to speak, of the paper.

1. Human Population Growth. $\dot{P} = \alpha P - \beta PC + \gamma C$.

1. The term, αP , assumes exponential growth absent other effects. This may or may not be reasonable. Reproduced in many ecology texts, *e.g.*, Hutchinson (1978), is the fit of pre-1940 American census data to the logistic equation which presumes an absolute cap (carrying capacity) on density. However, as everyone knows, the US population continued to grow thereafter, revealing the “fit” to be a fantasy. Today, human population growth rates are declining, and it has been suggested that the world’s population may stabilize at as little as 7-8.5 billion souls (Duncan and Foley, 2000).
2. The term, $-\beta PC$, presumes a negative effect of atmospheric carbon on human reproductive potential, *i.e.*, the *per capita* rate, which is $(\alpha - \beta C)$. Of this, there can be now doubt, though possibly not for the reasons usually given. Increasing $[\text{CO}_2]_{\text{atm}}$ may or may not prove harmful to humans via planetary warming – I do not wish to enter *that* discussion here. But the quasi-religious fervor now sweeping the world, if unchecked, will surely wreck the global economy (Byatt *et al.*, 2006; Yol and Yohe, 2006) with consequent reductions in the net rate (*per capita* births minus *per capita* deaths) of human increase. Indeed, one may observe more generally, that sustainability is linked by its advocates, *e.g.*, Desvaux (2007) to calls for substantial reductions in the human population (Kanie, 2006).
3. The term, γC , corresponds to augmentation, if positive, or diminution, if negative, of the total (as opposed to the *per capita*) rate of human increase. “The changing population,” Lonngren and Bai suggest, “will be affected by the change of the fossil fuels in the earth.” Fair enough. But on human time scales, C , being the concentration of atmospheric carbon, is not necessarily correlated with reserves in the ground. Nor is it obvious that such a relation

⁴ By “conventional,” I mean the range of possibilities encompassed by the models of Volterra (1926), Rosenzweig and MacArthur (1963) and MacArthur (1970).

should hold on the longer time scales to which Lonngren and Bai allude – their reference to dying populations being buried and decaying into fuel. Moreover, modeling the atmosphere on such time scales requires reference to geological processes, *e.g.*, Berner (2006) – certainly not a place to visit when formulating simple models.

As to modeling human activity millennia into the future, one can only marvel at the authors’ audacity. Our species is geologically recent. Uggg, the caveman, burned *renewable* fuels. And even though many of his descendants continue to do likewise (Martinez and Ebenhack, 2008), they, along with members of more technologically advanced societies, engage in activities that would have been difficult for Ugg to anticipate. Thus, while Pournelle *et al.* (1991) may have accurately prognosticated the evolution of climate politics, and possibly even climate science, some 20 years into the future, forecasting technological change over hundreds, much less millions, of years would seem quite impossible.

2. Atmospheric Carbon. $\dot{C} = -\delta C + \varepsilon CP$.

1. Regarding the term, $-\delta C$, Lonngren and Bai assert that “increasing amount[s] of carbon dioxide in the atmosphere will shield the solar radiation received at the earth and will therefore reduce the production of fossil fuels.” This seems at odds with the view, *e.g.*, Covey (1989) and McGuffie and Henderson-Sellers (2005), that the principal effect of increasing greenhouse gas concentrations is to reduce re-radiation of absorbed solar energy as infrared. If, as is widely believed, higher $[\text{CO}_2]_{\text{atm}}$ warms the climate, energy consumption may well decline. But this effect will be in proportion to the human population, in which case, the appropriate term is $-\delta PC$. Then atmospheric carbon increases or declines depending on whether $\delta < \varepsilon$ or vice versa. On the other hand, *regulatory responses* to increasing $[\text{CO}_2]_{\text{atm}}$, as endorsed, for example, by the Stern Review (Stearn *et al.*, 2006) and more extreme scenarios (Kanie, 2006), are arguably modeled in the way Lonngren and Bai propose. That is, what drives regulation is total atmospheric carbon, as opposed to carbon per individual. Similar considerations may apply to atmospheric scrubbing by the biosphere.
2. The term, $+\varepsilon PC$, models the generation of atmospheric carbon, with $[\text{CO}_2]_{\text{atm}}$ serving as a proxy variable for fuel.

A Modest Generalization. Lonngren and Bai point out that their model assumes replenishment of fossil fuels over geologic time. Relaxing this assumption, they suggest, leads to a different solution – no carbon and no people. In fact, reducing carbon replenishment via the parameter, ε , moves the C -isocline to the right. In the limit, $\varepsilon \rightarrow 0$, and $(P, C) \rightarrow (\infty, \alpha / \beta)$. This follows from the fact, noted above, that in the limit of large P , $C(P) \rightarrow \alpha / \beta$. As a real-world prediction, this is hard to swallow. Absent replacement sources of energy, a dearth of fossil fuel should reduce human populations to pre-Industrial Revolution levels, at the least. What is missing is a technology-dependent ceiling on human population (Deevey, 1960; Kremer, 1993). Additionally, Equations (1) use the variable, C , to represent both available fuel and atmospheric carbon. This suggests the utility of distinguishing $[\text{CO}_2]_{\text{atm}}$ from fuel, and fuel, from reserves in the ground. In this case, one might have something like the following:

$$\begin{aligned}
 \dot{R} &= A(P, F) - B(P, F, R); & \dot{F} &= B(P, F, R) - D(P, F); \\
 \dot{C} &= \varepsilon D(P, F) - H(C); & \dot{P} &= rP \left[1 - \frac{P}{K(F)} \right]
 \end{aligned}
 \tag{7}$$

- In the first equation, proven reserves, R , accumulate at rate, $A(P, F)$, and are extracted at rate, $B(P, F, R)$.
- In the second equation, available fuel, F , accumulates at rate, $B(P, F, R)$, and is consumed at rate, $D(P, F)$.
- In the third equation, atmospheric carbon, C , accumulates at rate $\varepsilon D(P, F)$, where ε is the fraction of consumed carbon released into the atmosphere. Correspondingly, C is depleted in proportion to its concentration – by natural processes such as photosynthesis, and possibly, by human technology.
- The final equation models human population growth, in which case the maximum supportable population, $K(F)$, is assumed to depend on available fuel.

One could, and probably should, further include the effects of carbon reduction schemes, in which case, the extraction and consumption functions, $B(\cdot)$ and $D(\cdot)$, as well as the parameters, ε , r and the carrying capacity, $K(F)$, would depend on C . The appropriate choice of the unspecified functions is, of course, beyond the scope of a brief communication

Fluctuating External Inputs. Even two variable models, when subjected to fluctuating external inputs, can manifest a wide range of behaviors. These include periodic and chaotic oscillations on time scales far longer than the period of the forcing – see King and Schaffer (1999) and Schaffer and Bronnikova (2007) for examples in ecology and epidemiology. In the case of carbon and climate, such forcing can result from variations in the solar constant, perhaps amplified by the mechanism suggested by Svensmark (1998, 2007) – but see Lockwood and Fröhlich (2007) who disagree. This sets up the possibility of climatic shifts of the sort described in the archaeological, *e.g.*, Arneborg *et al.* (1999) and paleoclimatological, *e.g.*, Andersen *et al.*, (2004) literature, not in terms of one to one correspondence with external drivers, but as *resonant responses* to “periodic” forcing.

Conclusion. Our analysis of Equations (1) is compatible with both the conventional wisdom and arguments against it. LB tells us that with $\gamma < 0$ (negative carbon effect), mankind perishes. What is not specified is the mechanism by which atmospheric carbon affects population growth. Those who endorse the “consensus” view will point to the *direct* effect of climate warming⁵ and its anticipated sequellae. By

⁵ For a minimally alarming estimate of climate sensitivity to $[\text{CO}_2]_{\text{atm}}$, see Chylek and Lohmann (2008).

way of contrast, skeptics will point to *indirect* effects – the untoward and unnecessary consequences of carbon-mitigating societal responses – to human reproduction and survival. Which view is correct cannot, of course, be determined by appeal to Equations (1) or (7). What these models *can* do is to focus attention on the question.

Acknowledgements. I thank Professors Lonngren and Bai for graciously agreeing to read and comment on the manuscript. The errors remain my own.

References.

Andersen, *et al.* 2004. High-resolution record of Northern Hemisphere climate extending into the last interglacial period. *Nature*. **431**: 147-151.

Arneborg, J., Heinemeier, J., Lynnerup, N., Nielsen, H. L., Rud, N, and A. E. Sveinbjörnsdóttir. 1999. Change of diet of the Greenland Vikings determined from stable carbon isotope analysis and ¹⁴C dating of their bones. *Radiocarbon*. **41**: 157-168.

Berner, R. A. 2006. GEOCARBSULF: A combined model for Phanerozoic atmospheric O₂ and CO₂. *Geochim. Cosmochim. Acta* **70**: 5653–5664.

Borges, J. L. 1998. On exactitude in science. P. 325, **In**, *Jorge Luis Borges, Collected Fictions*. (Transl. Hurley, H.) Penguin Books.

Byatt, I., Castles, I., Goklany, I. M., Henderson, D., Lawson, N., McKittrick, R., Morris, J., Peacock, A., Robinson, C. and R. Skidelsky. 2006. The Stern Review: A dual critique. Part II: Economic aspects. *World Economics*. **7**: 199-229.

Carter, R. M., de Freitas, C. R., Goklany, I. M., Holland, D. and R. S. Lindzen. 2006. The Stern Review: A Dual Critique. Part I: The science.

Chylek, P. and U. Lohmann, 2008. Aerosol radiative forcing and climate sensitivity deduced from the Last Glacial Maximum to Holocene transition. *Geophys. Res. Lett.* **35**: L04804, doi:10.1029/2007GL032759, 2008.

Covey, C. 1989. Mechanisms of climatic change. Pp. 11-33. **In**, Singer, F. (ed.) *Global Climate Change: Human and Natural Influences*. Paragon House, NY.

Deevey, E. S. .1960. The human population. *Scientific American*. **203**: 194–204.

Desvaux, M. 2007. The sustainability of human populations. *Significance*. **September** (2007). 102-107.

Duncan K. and D. K. Foley. 2000. Stabilization of human population through economic increasing returns. *Econom. Lett.* **68**: 309-317.

Guckenheimer, J. and P. Holmes. 1983. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer-Verlag, New York.

Kanie, N. 2006, Low carbon scenario toward 2050: Scenario development and its implication for policy measures. Global Environment Research Fund S3 (2050Research Project) FY2006 Research Results (S-3-2 Abstract) http://2050.nies.go.jp/material/H18-S-3/S-3-2/H18_S-3-2_abstract_e.pdf

Kremer, M. 1993. Population growth and technological change: One Million B.C. to 1990. *Quart. J. Econom.* **108**: 681-716.

King, A. A. and W. M. Schaffer. 1999. The rainbow bridge: Hamiltonian limits and resonance in predator-prey dynamics. *J. Math. Biol.* **39**: 439-469.

Lockwood, M. and C. Fröhlich. 2007. Recent oppositely directed trends in solar climate forcings and the global mean surface air temperature. *Proc. R. Soc. A.* **463**: 2447–2460.

Lonngren, K. E. and E.-W. Bai. 2008. On the global warming problem due to carbon dioxide. *Energy Policy*. In press.

MacArthur, R. H. 1970. Species packing and competitive equilibrium for many species. *Theoret. Pop. Biol.* **1**: 1-11.

Martinez, D.M. and B. W. Ebenhack, 2008. Understanding the role of energy consumption in human development through the use of saturation phenomena. *Energy Policy*, doi:10.1016/j.enpol.2007.12.016

McGuffie, K. and A. Henderson-Sellers. 2005. *A Climate Modeling Primer*. 3rd Edition. J. Wiley, NY.

Pournelle, J., Flynn, M. and L. Niven. 1991. *Fallen Angels*. Baen Books. NY.

Rosenzweig, M. L. and R. H. MacArthur. 1963. Graphical representation and stability conditions of predator prey interactions. *Amer. Natur.* **97**: 209-223.

Scafetta, N, and B. J. West. 2006. Phenomenological solar signature in 400 years of reconstructed Northern Hemisphere temperature record *Geophys. Res. Lett.* **33**: L17718, doi:10.1029/2006GL027142, 2006.

Schaffer, W. M. and T. V. Bronnikova. 2007. Parametric dependence in model epidemics. *J. Biol. Dynam.* **1**: 183-195.

Svensmark, H. 1998, Influence of cosmic rays on earth's climate. *Phs. Rev. Lett.* **85**: 5004-5007.

Svensmark, H. 2007 Cosmoclimatology: a new theory emerges. *Astron. Geophys.* **48**: 118–124.

Stern, N. S., Peters, V. Bakhshi, A. et al. 2006. *Stern Review: The Economics of Climate Change*, HM Treasury, London.

Tol, R. S. J. and G. W. Yohe. 2006. A Review of the Stern Review. *World Economics.* **7**: 233-250.

Volterra, V. 1926. Fluctuations in abundance of a species considered mathematically. *Nature*. **118**: 558-60.